

## Research Article

# Efficient Multichannel NLMS Implementation for Acoustic Echo Cancellation

Fredric Lindstrom,<sup>1</sup> Christian Schüldt,<sup>2</sup> and Ingvar Claesson<sup>2</sup>

<sup>1</sup> Konfiel AB, Research and Development, Box 268, 90106 Umea, Sweden

<sup>2</sup> Department of Signal Processing, Blekinge Institute of Technology, 37225 Ronneby, Sweden

Received 31 May 2006; Revised 9 November 2006; Accepted 14 November 2006

Recommended by Kutluyil Dogancay

An acoustic echo cancellation structure with a single loudspeaker and multiple microphones is, from a system identification perspective, generally modelled as a single-input multiple-output system. Such a system thus implies specific echo-path models (adaptive filter) for every loudspeaker to microphone path. Due to the often large dimensionality of the filters, which is required to model rooms with standard reverberation time, the adaptation process can be computationally demanding. This paper presents a selective updating normalized least mean square (NLMS)-based method which reduces complexity to nearly half in practical situations, while showing superior convergence speed performance as compared to conventional complexity reduction schemes. Moreover, the method concentrates the filter adaptation to the filter which is most misadjusted, which is a typically desired feature.

Copyright © 2007 Fredric Lindstrom et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

## 1. INTRODUCTION

Acoustic echo cancellation (AEC) [1, 2] is used in teleconferencing equipment in order to provide high quality full-duplex communication. The core of an AEC solution is an adaptive filter which estimates the impulse response of the loudspeaker enclosure microphone (LEM) system. Typical adaptive algorithms for the filter update procedure in the AEC are the least mean square, normalized least mean square (LMS, NLMS) [3], affine projection (AP), and recursive least squares (RLS) algorithms [4]. Of these, the NLMS-based algorithms are popular in industrial implementations, thanks to their low complexity and finite precision robustness.

Multimicrophone solutions are frequent in teleconferencing equipment targeted for larger conference rooms. This paper considers a system consisting of one loudspeaker and three microphones. The base unit of the system contains the loudspeaker and one microphone and it is connected to two auxiliary expansion microphones, as shown in Figure 1. Such multimicrophone system constitutes a single-input multiple-output (SIMO) multichannel system with several system impulse responses to be identified, Figure 2. Thus, the signal processing task can be quite computational demanding.

Several methods for computational complexity reduction of the LMS/NLMS algorithms have been proposed and ana-

lyzed, for example, [5–14]. In this paper a related low complexity algorithm for use in a multimicrophone system is proposed.

## 2. COMPLEXITY REDUCTION METHODS

The LEM system can be modelled as a time invariant linear system,  $\mathbf{h}(k) = [h_0(k), \dots, h_{N-1}(k)]^T$ , where  $N - 1$  is the order of the finite impulse response (FIR) model [11] and  $k$  is the sample index. Thus, the desired (acoustic echo) signal  $d(k)$  is given by  $d(k) = \mathbf{h}(k)^T \mathbf{x}(k)$ , where  $\mathbf{x}(k) = [x(k), \dots, x(k - N + 1)]^T$  and  $x(k)$  is the input (loudspeaker) signal. The measured (microphone) signal  $y(k)$  is obtained as  $y(k) = d(k) + n(k)$ , where  $n(k)$  is near-end noise. Assuming an adaptive filter  $\hat{\mathbf{h}}(k)$  of length  $N$  is used, that is,  $\hat{\mathbf{h}}(k) = [\hat{h}_0(k), \dots, \hat{h}_{N-1}(k)]^T$ , the NLMS algorithm is given by

$$e(k) = y(k) - \hat{d}(k) = y(k) - \mathbf{x}(k)^T \hat{\mathbf{h}}(k), \quad (1)$$

$$\beta(k) = \frac{\mu}{\|\mathbf{x}(k)\|^2 + \epsilon}, \quad (2)$$

$$\hat{\mathbf{h}}(k + 1) = \hat{\mathbf{h}}(k) + \beta(k)e(k)\mathbf{x}(k),$$

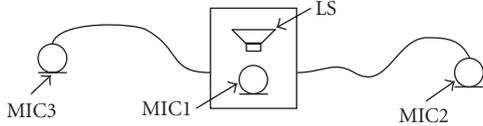


FIGURE 1: AEC unit with expansion microphones.

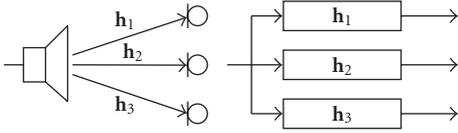


FIGURE 2: Schematic picture over multimicrophone system modelled as a single-input multiple-output system.

where  $\hat{d}(k)$  is the estimated echo,  $e(k)$  the error (echo cancelled) signal,  $\beta(k)$  the step-size,  $\|\mathbf{x}(k)\|^2 = \mathbf{x}(k)^T \mathbf{x}(k)$  the squared Euclidian norm,  $\mu$  the step size control parameter, and  $\epsilon$  a regularization parameter [4].

Low-complexity periodical and partial updating schemes reduce the computational complexity of the LMS/NLMS by performing only a part of the filtering update, (2). The periodic NLMS performs the filter update only at periodical sample intervals. This updating can be distributed over the intermediate samples [5]. The sequential NLMS updates only a part of the  $N$  coefficients at every sample in a sequential manner [5]. Several methods for choosing which coefficients to update at what sample instant have been proposed, for example, choosing a subset containing the largest coefficients in the regressor vector [6], low-complexity version of largest regressor vector coefficient selection [7], block-based regressor vector methods [8, 9], and schemes based on randomization in the update procedure [10]. The updating can also be based on assumptions of the unknown plant [11, 12]. Another approach of omitting updates is possible in algorithms where the step size is zero for a large number of updates [13, 14].

In a SIMO-modelled  $M$  microphone system, there are  $M$  adaptive filters  $\hat{\mathbf{h}}_m(k)$  with  $m \in \{1, \dots, M\}$ , to be updated at each sample, that is,

$$\hat{\mathbf{h}}_m(k+1) = \hat{\mathbf{h}}_m(k) + \frac{\mu e_m(k) \mathbf{x}(k)}{\|\mathbf{x}(k)\|^2 + \epsilon} \quad m = 1, \dots, M, \quad (3)$$

see Figure 2 for an example with  $M = 3$ . The updating scheme proposed in this paper explores the possibility of choosing between the different update equations based on comparison between the  $M$  different error signals  $e_m(k)$ .

### 3. THE PROPOSED ALGORITHM

An adaptive linear filtering process can generally be divided in two parts the *filtering* (1) and the *adaptation* (2). In an echo cancellation environment, the filtering part generally is performed at every sample instant in order to produce a constant audio stream. Although it is most often efficient

TABLE 1: Example to illustrate the matrix  $\mathbf{E}(k)$ .

Sample index	Filter 1	Filter 2	Filter 3
$k$	$e_1(k)$	$e_2(k)$	$e_3(k)$
$k-1$	Update	$e_2(k-1)$	$e_3(k-1)$
$k-2$	X	$e_2(k-2)$	Update
$k-3$	X	Update	X

(in terms of convergence) to perform filter updating at every sample instant, it is not necessary. In practice, this might not even be possible due to complexity issues. This especially applies to acoustic echo cancellation environments where the dimension of the system filters is large.

One approach in a  $M$ -microphone system is to update only one adaptive filter every sample in a round-robin manner, that is, periodic NLMS. This also ensures equal (for all filters) and predictable convergence since the update occurrences are deterministic. The disadvantage is that convergence is slow.

This paper proposes another updating method which instead updates the filter with the largest output error. To illustrate the method, assume that  $M = 3$  (3 adaptive filters), the present sample index is  $k$ , and filter 1 was updated at sample index  $k-1$ , filter 3 at  $k-2$ , and filter 2 at  $k-3$ , as illustrated in Table 1. Thus, the available errors that can be used in the update at the present sample index  $k$  are  $e_1(k)$  for filter 1,  $e_2(k)$ ,  $e_2(k-1)$  and  $e_2(k-2)$  for filter 2, and  $e_3(k)$  and  $e_3(k-1)$  for filter 3. For example, the error  $e_1(k-2)$  cannot be used since it is related to the configuration of filter 1 prior to the latest update. From the available errors, the algorithm chooses the error with the largest magnitude and then performs the corresponding update (compare with (6) and (7) below).

An algorithm for the method is as follows. After filtering all  $M$ -output channels according to (1), the output errors from all filters are inserted in a  $L \times M$  matrix

$$\mathbf{E}(k) = \begin{pmatrix} e_1(k) & e_2(k) & e_3(k) & \dots & e_M(k) \\ & & \mathbf{E}(k-1) & & \end{pmatrix}, \quad (4)$$

where  $M$  is the number of adaptive filters (channels) and  $L$  determines the number of previous samples to consider. The  $L-1 \times M$  matrix  $\mathbf{E}(k-1)$  consists of the  $L-1$  upper rows of  $\mathbf{E}(k-1)$ , that is,

$$E(l+1, m, k) = E(l, m, k-1) \quad l = 1, \dots, L-1, \quad m = 1, \dots, M, \quad (5)$$

where  $l$  and  $m$  denote row and column indexes, respectively, and  $E(l, m, k)$  is the element at row  $l$  and column  $m$  in  $\mathbf{E}(k)$ .

The decision of which filter to update and with what output error (and corresponding input vector) is determined by the element in  $\mathbf{E}(k)$  with maximum absolute value,

$$e_{\max}(k) = \max_{l,m} |E(l, m, k)| \quad l = 1, \dots, L, \quad m = 1, \dots, M. \quad (6)$$

The row and column indexes of the element in  $\mathbf{E}(k)$  with the maximum absolute value are denoted  $l_{\max}(k)$  and  $m_{\max}(k)$ .

For clarity of presentation, the sample index is omitted, that is,  $l_{\max} = l_{\max}(k)$  and  $m_{\max} = m_{\max}(k)$ .

The filter corresponding to the row index  $m_{\max}$ , that is, the filter  $\hat{\mathbf{h}}_{m_{\max}}(k)$ , is then updated with

$$\hat{\mathbf{h}}_{m_{\max}}(k+1) = \hat{\mathbf{h}}_{m_{\max}}(k) + \frac{\mu e_{m_{\max}}(k) \mathbf{x}(k - l_{\max} + 1)}{\|\mathbf{x}(k - l_{\max} + 1)\|^2 + \epsilon}. \quad (7)$$

This filter update of filter  $\hat{\mathbf{h}}_{m_{\max}}(k)$  will make the error elements  $E(l, m_{\max}, k)$ ,  $l = 1, \dots, L$  obsolete, since these are errors generated by  $\hat{\mathbf{h}}_{m_{\max}}(k)$  prior to the update. Consequently, to avoid future erroneous updates, these elements should be set to 0, that is, set

$$E(l, m_{\max}, k) = 0 \quad \text{for } l = 1, \dots, L. \quad (8)$$

An advantage over periodic NLMS is that the proposed structure does not limit the update to be based on the current input vector  $\mathbf{x}(k)$ , but allows updating based on previous input vectors as well, since the errors not yet used for an update are stored in  $\mathbf{E}(k)$ . Further, largest output-error update will concentrate the updates to the corresponding filter. This is normally a desired feature in an acoustic echo cancellation environment with multiple microphones. For example, consider the setup in Figure 1 with all adaptive filters fairly converged. If then one of the microphones is dislocated, this results in an echo-path change for the corresponding adaptive filter. Naturally, it is desired to concentrate all updates to this filter.

#### 4. ANALYSIS

In the previously described scenario, where several input vectors are available but only one of them can be used for adaptive filter updating (due to complexity issues), it might seem intuitive to update with the input vector corresponding to the largest output error magnitude. In this section, it is shown analytically that, under certain assumptions, choosing the largest error maximizes the reduction.

The error deviation vector for the  $m$ th filter  $\mathbf{v}_m(k)$  is defined as  $\mathbf{v}_m(k) = \mathbf{h}_m(k) - \hat{\mathbf{h}}_m(k)$ , and the mean-squared deviation as  $\mathcal{D}(k) = \mathbb{E}\{\|\mathbf{v}_m(k)\|^2\}$ , where  $\mathbb{E}\{\cdot\}$  denotes expectation [4]. Assume that no near-end sound is present,  $n(k) = 0$ , and no regularization is used,  $\epsilon = 0$ , and that the errors available for updating filter  $m$  are  $e_m(k - l_m)$  with  $l_m = 0, \dots, L_m$  and  $L_m < L$ , that is, the available errors in matrix  $\mathbf{E}(k)$  that correspond to filter  $m$ . Updating filter  $m$  using error  $e_m(k - l_m)$  gives

$$\|\mathbf{v}_m(k+1)\|^2 = \|\mathbf{v}_m(k) - \beta(k)e_m(k - l_m)\mathbf{x}(k - l_m)\|^2 \quad (9)$$

and by using

$$e_m(k - l_m) = \mathbf{x}(k - l_m)^T \mathbf{v}_m(k) = \mathbf{v}_m(k)^T \mathbf{x}(k - l_m) \quad (10)$$

in (9), the following is obtained:

$$\|\mathbf{v}_m(k+1)\|^2 = \mathbf{v}_m(k)^T \mathbf{v}_m(k) - \frac{(2\mu - \mu^2)}{\|\mathbf{x}(k - l_m)\|^2} e_m^2(k - l_m). \quad (11)$$

Thus, the difference in mean-square deviation from one sample to the next is given by

$$\mathcal{D}_m(k+1) - \mathcal{D}_m(k) = -(2\mu - \mu^2) \mathbb{E} \left\{ \frac{e_m^2(k - l_m)}{\|\mathbf{x}(k - l_m)\|^2} \right\}, \quad (12)$$

which corresponds to a reduction under the assumption that  $0 < \mu < 2$ .

Further, assuming small fluctuations in the input energy  $\|\mathbf{x}(k)\|^2$  from one iteration to the next, that is, assuming

$$\|\mathbf{x}(k)\|^2 = \|\mathbf{x}(k-1)\|^2 = \dots = \|\mathbf{x}(k - L_m + 1)\|^2, \quad (13)$$

gives [4],

$$\mathcal{D}_m(k+1) - \mathcal{D}_m(k) = -(2\mu - \mu^2) \frac{\mathbb{E}\{e_m^2(k - l_m)\}}{\mathbb{E}\{\|\mathbf{x}(k)\|^2\}}. \quad (14)$$

The total reduction  $r(k)$  in deviation, considering all  $M$  filters is thus

$$r(k) = \sum_{m=1}^M \mathcal{D}_m(k+1) - \mathcal{D}_m(k). \quad (15)$$

Only one filter is updated each time instant. Assume error  $E(l, m, k)$  is chosen for the update. Then  $r(k)$  is given by

$$r(k) = -(2\mu - \mu^2) \frac{\mathbb{E}\{E^2(l, m, k)\}}{\mathbb{E}\{\|\mathbf{x}(k)\|^2\}}. \quad (16)$$

From (16), it can be seen that the reduction is maximized if  $e_{\max}(k)$ , (see (16)), is chosen for the update, that is, as done in the proposed algorithm.

The proposed algorithm can be seen as a version of the periodic NLMS. Analysis of convergence, stability, and robustness for this branch of (N)LMS algorithms are provided in, for example, [5, 15].

#### 5. COMPLEXITY AND IMPLEMENTATION

The algorithm proposed in this paper is aimed for implementation in a general digital signal processor (DSP), typically allowing multiply add and accumulate arithmetic operations to be performed in parallel with memory reads and/or writes (e.g., [16]). In such a processor, the filtering operation can be achieved in  $N$  instructions and the NLMS update will require  $2N$  instructions. Both the filtering and the update require two memory reads, one addition and one multiplication per coefficient, which can be performed by the DSP in one instruction. However, the result from the filter update is not accumulated but it needs to be written back to memory. Therefore, the need for two instructions per coefficient for the update operation.

Suppose an  $M$ -channel system with the same number of adaptive filters, all with the length of  $N$ . The standard NLMS updating thus requires  $3MN$  DSP instructions.

Updating the matrix  $\mathbf{E}(k)$ , (4), can be implemented using circular buffering and thus requires only  $M$  store instructions (possible pointer modifications disregarded), while clearing of  $\mathbf{E}(k)$ , (8), takes a maximum of  $L$  instructions (also disregarding possible pointer modifications). Searching for the maximum absolute valued element in  $\mathbf{E}(k)$ , (6), requires a maximum of  $2LM$  instructions ( $LM$  abs-instructions and  $LM$  max-instructions). The parameter  $\|\mathbf{x}(k)\|^2$  can be calculated very efficient through recursion, that is,

$$\|\mathbf{x}(k)\|^2 = \|\mathbf{x}(k-1)\|^2 + x^2(k) - x^2(k-N), \quad (17)$$

and its computational complexity can be disregarded in this case.

All together, this means that the number of DSP instructions required for the proposed solution can be approximated with

$$MN + M + L + 2ML + 2N. \quad (18)$$

For acoustic echo cancellation,  $N$  is generally quite large ( $>1000$ ) due to room reverberation time. In this case, we typically have  $N \gg L$  and  $N \gg M$ , which means that (18) is approximately  $N(M+2)$ . The complexity reduction in comparison with standard NLMS updating is then

$$\frac{M+2}{3M}, \quad (19)$$

which for  $M=3$  gives a complexity reduction of nearly a half (5/9). For higher values of  $M$ , the reduction is even larger. Further reduction in complexity can also be achieved if updates are performed say every other or every third sample.

## 6. SIMULATIONS

The performance of the proposed method was evaluated through simulations with speech as input signal. Three impulse responses ( $\mathbf{h}_1$ ,  $\mathbf{h}_2$ , and  $\mathbf{h}_3$ ), shown in Figure 3, all of length  $N=1800$  were measured with three microphones, according to the constellation in Figure 1, in a normal office. The acoustic coupling between the loudspeaker and the closest microphone, AC1, was manually normalized to 0 dB and the coupling between the loudspeaker and the second and third microphones, AC2 and AC3, were then estimated to  $-6$  dB and  $-7$  dB, respectively. Thus,  $10 \log_{10}(\|\mathbf{h}_2\|^2/\|\mathbf{h}_1\|^2) = -6$  dB and  $10 \log_{10}(\|\mathbf{h}_3\|^2/\|\mathbf{h}_1\|^2) = -7$  dB.

Output signals  $y_1(k)$ ,  $y_2(k)$ , and  $y_3(k)$  were obtained by filtering the input signal  $x(k)$  with the three obtained impulse responses and adding noise,

$$\begin{aligned} y_1(k) &= \mathbf{x}(k)^T \mathbf{h}_1 + n_1(k), \\ y_2(k) &= \mathbf{x}(k)^T \mathbf{h}_2 + n_2(k), \\ y_3(k) &= \mathbf{x}(k)^T \mathbf{h}_3 + n_3(k). \end{aligned} \quad (20)$$

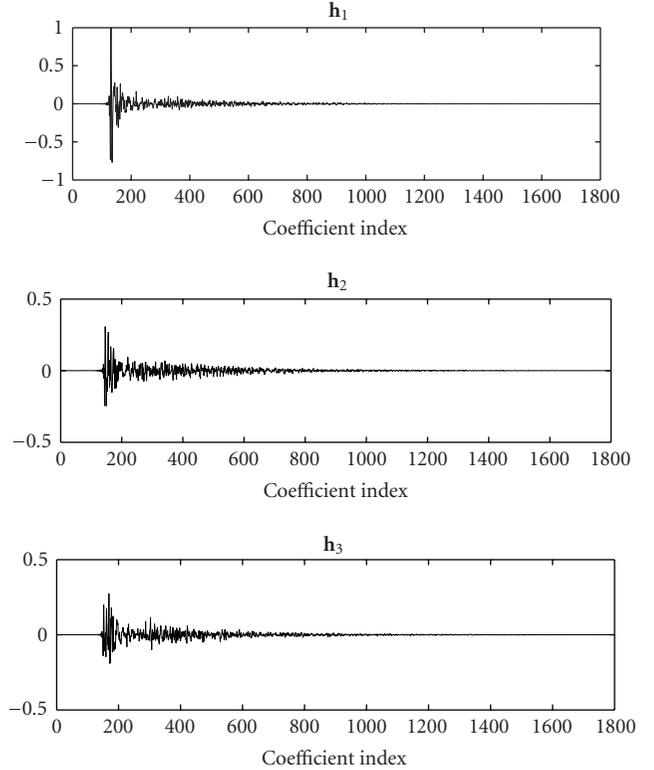


FIGURE 3: Impulse responses used in the simulations.

The noise sources  $n_1(k)$ ,  $n_2(k)$ , and  $n_3(k)$  were independent, but had the same characteristics (bandlimited flat spectrum). Echo-to-noise ratio was approximately 40 dB for microphone 1 and 34 dB and 33 dB for microphones 2 and 3, respectively.

In the simulations four low-complexity methods of similar complexity were compared; the periodic (N)LMS [5], random NLMS (similar to SPU-LMS [10]) selecting which filter to be updated in a stochastic manner (with all filters having equal probability of an update), M-Max NLMS [6], and the proposed NLMS. The performance of the full update NLMS is also shown for comparison. The periodic NLMS, random NLMS, and the proposed method limit the updates to one whole filter at each time interval, while M-Max NLMS instead updates all filters but only does this for a subset (1/3 in this case) of all coefficients. However, since M-Max NLMS requires sorting of the input vectors, the complexity for this method is somewhat larger ( $2 \log_2 N + 2$  comparisons and  $(N-1)/2$  memory transfers [9]). Zero initial coefficients were used for all filters and methods. The result is presented in Figure 4, where the normalized filter mismatch, calculated as

$$10 \log_{10} \left( \frac{\|\mathbf{h}_m - \hat{\mathbf{h}}_m(k)\|^2}{\|\mathbf{h}_m\|^2} \right) \quad m = 1, 2, 3, \quad (21)$$

for the three individual filters and solutions are presented. Of the four variants with similar complexity, the proposed method is clearly superior to the conventional periodic

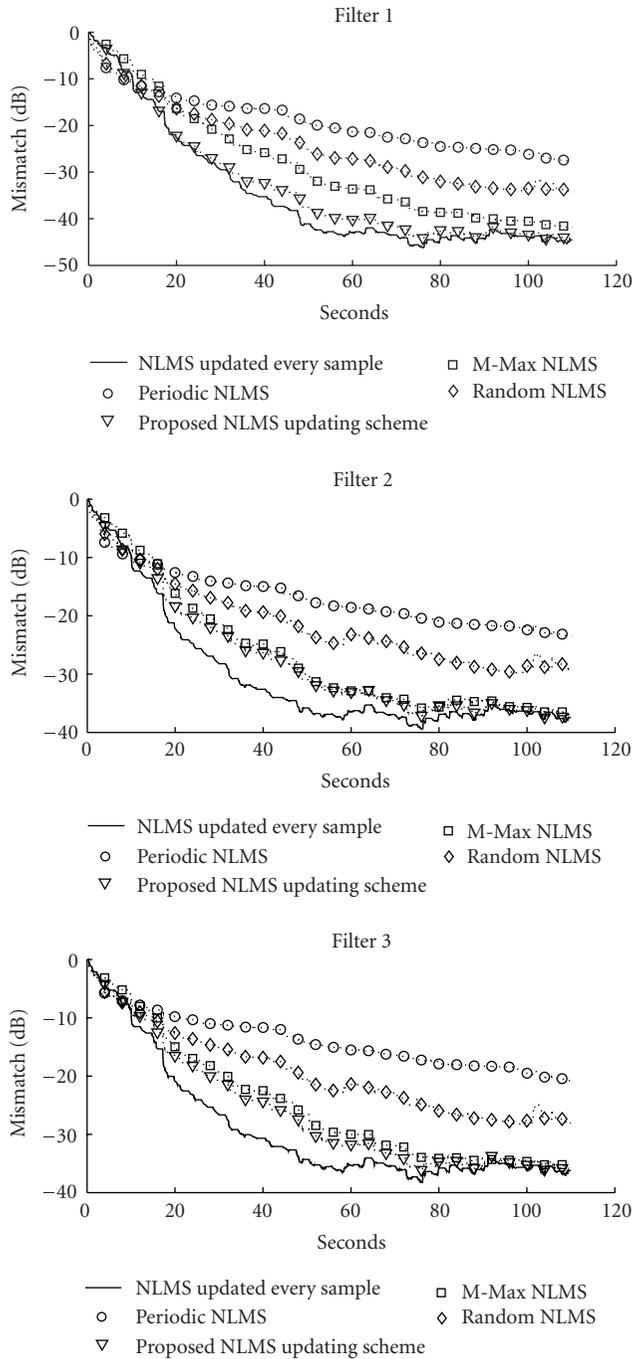


FIGURE 4: Mismatch for the the evaluated methods.

NLMS and also to the random NLMS. The performance of the M-Max NLMS and the proposed solution is comparable, although the proposed solution performs better or equal for all filters.

The algorithm automatically concentrates computational resources to filters with large error signals. This is demonstrated in Figure 5, where filter 2 undergoes an echo-path change, that is, a dislocation of the microphone. In Figure 5,

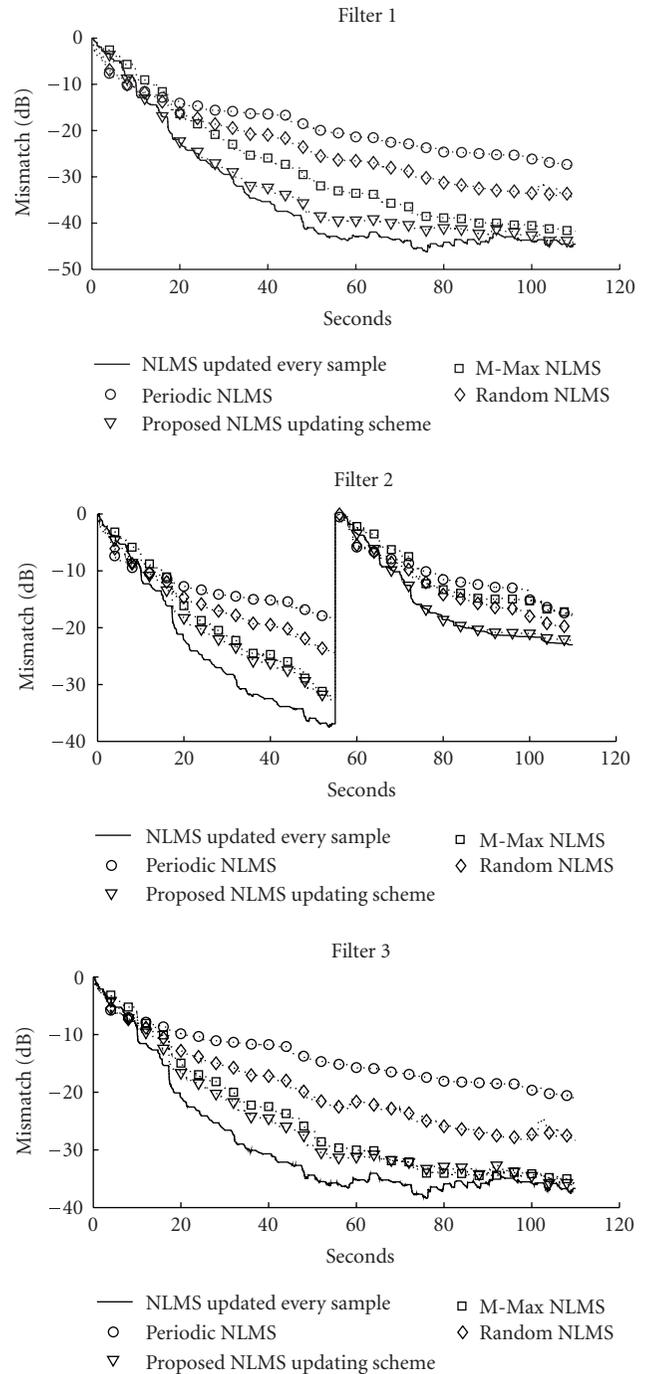


FIGURE 5: Mismatch for the the evaluated methods, where an echo-path change occurs for filter 2 after 55 seconds.

it can be seen that the proposed algorithm basically follows the curve of the full update NLMS immediately after the echo-path changes.

If one specific microphone is subject to an extreme acoustic situation, for example, it is placed in another room or placed immediately next to a strong noise source, there is a risk of “getting stuck,” that is, the corresponding filter has large output error for all input vectors and thus is updated all

the time. This problem can be reduced by setting a limit on the lowest rate of updates for a filter, that is, if filter  $m$  has not been updated for the last  $U$  samples it is forced to update the next iteration. However, this does not resolve the issue optimally. A more sophisticated method is to monitor the echo reduction of the filters and bypass or reduce the resources allocated to filters not providing significant error reduction. Implementing these extra functions will of course add complexity.

## 7. CONCLUSIONS

In an acoustic multichannel solution with multiple adaptive filters, the computation power required to update all filters every sample can be vast. This paper has presented a solution which updates only one filter every sample and thus significantly reduces the complexity, while still performing well in terms of convergence speed. The solution also handles echopath changes well, since the most misadjusted filter gets the most computation power, which often is a desirable feature in practice.

## ACKNOWLEDGMENT

The authors would like to thank the Swedish Knowledge Foundation (KKS) for funding.

## REFERENCES

- [1] E. Hänsler and G. Schmidt, *Acoustic Echo and Noise Control: A Practical Approach*, John Wiley & Sons, New York, NY, USA, 2004.
- [2] M. M. Sondhi, "An adaptive echo canceler," *Bell System Technical Journal*, vol. 46, no. 3, pp. 497–510, 1967.
- [3] B. Widrow and S. D. Stearns, *Adaptive Signal Processing*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1985.
- [4] S. Haykin, *Adaptive Filter Theory*, Prentice-Hall, Englewood Cliffs, NJ, USA, 4th edition, 2002.
- [5] S. C. Douglas, "Adaptive filters employing partial updates," *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 44, no. 3, pp. 209–216, 1997.
- [6] T. Aboulnasr and K. Mayyas, "Complexity reduction of the NLMS algorithm via selective coefficient update," *IEEE Transactions on Signal Processing*, vol. 47, no. 5, pp. 1421–1424, 1999.
- [7] P. A. Naylor and W. Sherliker, "A short-sort M-Max NLMS partial-update adaptive filter with applications to echo cancellation," in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP '03)*, vol. 5, pp. 373–376, Hong Kong, April 2003.
- [8] K. Dogaçay and O. Tanrikulu, "Adaptive filtering algorithms with selective partial updates," *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 48, no. 8, pp. 762–769, 2001.
- [9] T. Schertler, "Selective block update of NLMS type algorithms," in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP '98)*, vol. 3, pp. 1717–1720, Seattle, Wash, USA, May 1998.
- [10] M. Godavarti and A. O. Hero III, "Partial update LMS algorithms," *IEEE Transactions on Signal Processing*, vol. 53, no. 7, pp. 2382–2399, 2005.
- [11] E. Hänsler and G. Schmidt, "Single-channel acoustic echo cancellation," in *Adaptive Signal Processing*, J. Benesty and Y. Huang, Eds., Springer, New York, NY, USA, 2003.
- [12] S. M. Kuo and J. Chen, "Multiple-microphone acoustic echo cancellation system with the partial adaptive process," *Digital Signal Processing*, vol. 3, no. 1, pp. 54–63, 1993.
- [13] S. Gollamudi, S. Kapoor, S. Nagaraj, and Y.-F. Huang, "Set-membership adaptive equalization and an updator-shared implementation for multiple channel communications systems," *IEEE Transactions on Signal Processing*, vol. 46, no. 9, pp. 2372–2385, 1998.
- [14] S. Werner, J. A. Apolinario Jr., M. L. R. de Campos, and P. S. R. Diniz, "Low-complexity constrained affine-projection algorithms," *IEEE Transactions on Signal Processing*, vol. 53, no. 12, pp. 4545–4555, 2005.
- [15] W. A. Gardner, "Learning characteristics of stochastic-gradient-descent algorithms: a general study, analysis, and critique," *Signal Processing*, vol. 6, no. 2, pp. 113–133, 1984.
- [16] *ADSP-BF533 Blackfin processor hardware reference*, Analog Devices, Norwood, Mass, USA, 2005.